ANEXO 2

Possible verification of the linear correlation between the a and b Gutenberg-Richter parameters in the GR-law by using seismic data of some regions of the Mexican Pacific coast

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Abstract. In a previous study, our group proposed a positive correlation between the parameters a and b of the Gutenberg-Richter relation. That work provided evidence based on real global seismic catalogs, synthetic catalogs (generated from a spring-block model), and an analytical demonstration. In the present paper, we introduce new results obtained from analyzing an earthquake catalog covering the past 24 years in five seismic provinces along the Mexican Pacific coast, as well as the entire coastal zone. This analysis produced 24 pairs of Gutenberg-Richter parameters for each region. The results confirm the previously established positive correlation.

1. Introduction

In 2018, our group published an article [1] where we showed that the Gutenberg-Richter (GR) parameters a and b in the following equation: $log\dot{N}(M) = a - bM$, where $\dot{N}(M)$ is the number of earthquakes with magnitude larger than M per year [2, 3] and a and b, are linearly correlated. Curiously, as far as we know this issue has not received too much attention in the current seismic literature. The demonstration of such linear correlation was motivated on one hand for a paper by Bayrak et al. [4] where they reported (a, b) data from 27 active seismic areas around the Earth. We made a linear regression of the 27 pairs (a, b) reported in [4] (see figure 1 of [1]), obtaining:

$$a = (3.96 \pm 0.49)b + (3.62 \pm 0.38), \quad (1)$$

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On the other hand, we made around 150 catalogs of synthetic seisms obtained with a spring-block model due to Olami, Feder and Christensen (OFC) [5]. This model emulates a subduction seismic zone by means of relative movement of two horizontal plates in such a way that in the inferior plate a set of blocks are distributed in sites (i, j) of a square lattice where $i = 1,2, \dots, L$ and $j = 1,2, \dots, L$, being L the size of the lattice. All the blocks are interconnected by springs of elastic constants k_1, k_2 and k_L (see figure 2 of [1]). Besides, the blocks are connected by means of springs of constant k_L with a superior plate that moves with a small velocity v with respect to the inferior plate. Through the vertical springs mechanical energy is transmitted to the blocks of inferior plate. Each block only can give energy to its neighbor blocks only if it reaches a predetermined threshold value. If the neighbor blocks reach also the threshold value then it gives all the energy units to the corresponding neighbors developing a chain reaction until all blocks are below the threshold value and then chain reaction ends. The number of perturbed blocks from the beginning to the end of the chain reaction is the size of a synthetic event.

The described behavior of the previous array of blocks constitutes a typical case of self-organized criticality (SOC) [1, 5, 6-10] where the complex SOC systems are dominated by short-range interactions and without any characteristic scale. Since 1997 [11] Geller et al. confirmed the idea that dynamical behavior of Earth's crust corresponds reasonably to a SOC complex system. In first place, because many spring-block SOC models nicely reproduce the GR-law of seismicity. In addition, these SOC models reproduce also Utsu-Seki law [12] on rupture areas of an earthquake and the so-called Ruff-Kanamori diagram where a positive bilinear correlation between the magnitude and both the age of subducted tectonic plates and the relative velocity between the two plates involved in the subduction process was empirically suggested [13]. In 2020, our group emulated the Ruff-Kanamori diagram by using a SOC spring-block model with very reasonably results. As we said, we obtained around 150 synthetic catalogs by means of the OFC spring-block SOC model. By using the a and b values of the GR relationships obtained by synthetic catalogs we get excellent linear correlations between the a and b-GR parameters (see figure 5 of [1]). In reference [1] we also presented an analytical demonstration of the linear correlation between the a and b-GR parameters. This demonstration was based in the Utsu-Seki law:

$$\log S = M - 4 \qquad (2),$$

S in square kilometers and log is a base 10 logarithm. And also in the Aki relation [14]:

$$\dot{N} = CS^{\left(\frac{-D}{2}\right)} \qquad (3),$$

where S is the area of the earthquake rupture, D is the fractal dimension and C is a constant. Moreover, we used a second Aki relation given by:

$$D \approx 2b \qquad (4).$$

Then by combining (3) and (4) one immediately obtains:

$$\dot{N} = CS^{-b} \qquad (5),$$

by taking logarithms in both sides of this equation, one arrives to

$$\log \dot{N} = \log C - b \log S \qquad (6).$$

By substituting (2) in (6) and equating with the GR relation then, one immediately obtains:

$$a = 4b + \log C \quad (7).$$

Interestingly, when we compare this result with (1), which was obtained from 27 worldwide seismic catalogs [4], the coefficient of the b value in that expression is 3.957; that is, a value very close to the coefficient of the b value in (7), which is 4. The $\log C$ value in (1) is $\log C = 3.62$. In reference [1] we present a discussion on this value by using arguments derived from [16], which is a study on the fractal geometry and seismicity in the Mexican subduction zone and we arrived to $\log C = 3.962$; that is, a value no so far from 3.62. Thus, our analytical result is reasonably accordance with field measurements of a and b. The present work is organized as follows: In section 1, we present the introduction; in section 2, we present the possible empirical verification of (7) and finally, in section 3, we present our conclusions.

2. Possible empirical verification of a = 4b + log C

Recently, we have calculated GR relationships for five seismic provinces of Mexico along the Mexican Pacific coast near from the so-called Pacific trench which is the border between the Cocos plate and the Northamerican plate corresponding to Mexico. The considered provinces are Jalisco, Michoacán, Guerrero, Oaxaca and Chiapas (see figure 1). These Mexican states are zones of high seismicity. The GR relationships were obtained by using seismic catalogs from January 1st of 2000 until December 31st of 2023 taken from the USGS archives. To investigate the possible verification of the positive correlation between the a and b-GR parameters given by (7), we calculate the GR relationship for each of the five mentioned provinces. We choose as threshold magnitude (magnitude of completeness) $M_0 = 3.8$ and all data are reported in terms of s-waves; that is, M_s magnitudes. Our results were the following ones:

2.1. Jalisco case

The considered area from Jalisco is limited for the coordinates [17.5º-20.1º N, 103.55º-105.5 W] and depth $h \leq 60$ km, where occurred 101 seisms with $M_s \geq 3.8$ along 24 years. In this region we have data for 22 years. For each of these years we obtained a GR relationship given by: $\log \dot{N}(M) = a - bM$. In figure 2(a), we show the behaviour of the linear correlation (7). The obtained relation is $a = 3.7516\, b + 0.6725$ (with $R = 0.9948$). As we can see the coefficient of the b-GR parameter is not so far from 4 as reported for (7) and also by Bayrak et al. [4] in (1).

2.2. Michoacán case

The considered area from Michoacán is limited by the coordinates [16.4º-19.0º N, 101.5-103.55 W] and depth $h \leq 60$ km, where occurred 120 seisms with $M_s \geq 3.8$ along 24 years. In this region we have data for 19 years. For each of these years we obtained a GR relationship given by: $\log \dot{N}(M) = a - bM$. In figure 2(b), we show the behaviour of the linear correlation (7). The obtained relation is $a = 3.9967\, b + 0.4259$ (with $R = 0.9916$). Also, in this case we observed that the coefficient of b-GR is very near to 4.

2.3. Guerrero case

The studied area from Guerrero is limited by the coordinates [15.5º, 18.0º N, 98.4º-101.5º W] and depth $h \leq 60$ km, where occurred 441 seisms with $M_s \geq 3.8$ along 24 years. In this region we have data for 24 years. For each of these years we obtained a GR relationship given by: $\log \dot{N}(M) = a - bM$. In figure 2(c), we show the behaviour of the linear correlation (7). The obtained relation is $a =$

$4.0842\,b + 0.9242$ (with $R = 0.973$). Also, in this case we observed that the coefficient of b-GR is very near to 4.

2.4. Oaxaca case

The studied area from Oaxaca is limited by the coordinates [14.4°-17.4° N, 95.0°-98.4° W] and depth $h \leq 60$ km, where occurred 501 seisms with $M_s \geq 3.8$ along 24 years. In this region we have data for 24 years. For each of these years we obtained a GR relationship given by: $\log \dot{N}(M) = a - bM$. In figure 2(d), we show the behaviour of the linear correlation (7). The obtained relation is $a = 3.6584\,b + 1.3038$ (with $R = 0.9934$). Also, in this case we observed that the coefficient of b-GR is not so far to 4.

2.5. Chiapas case

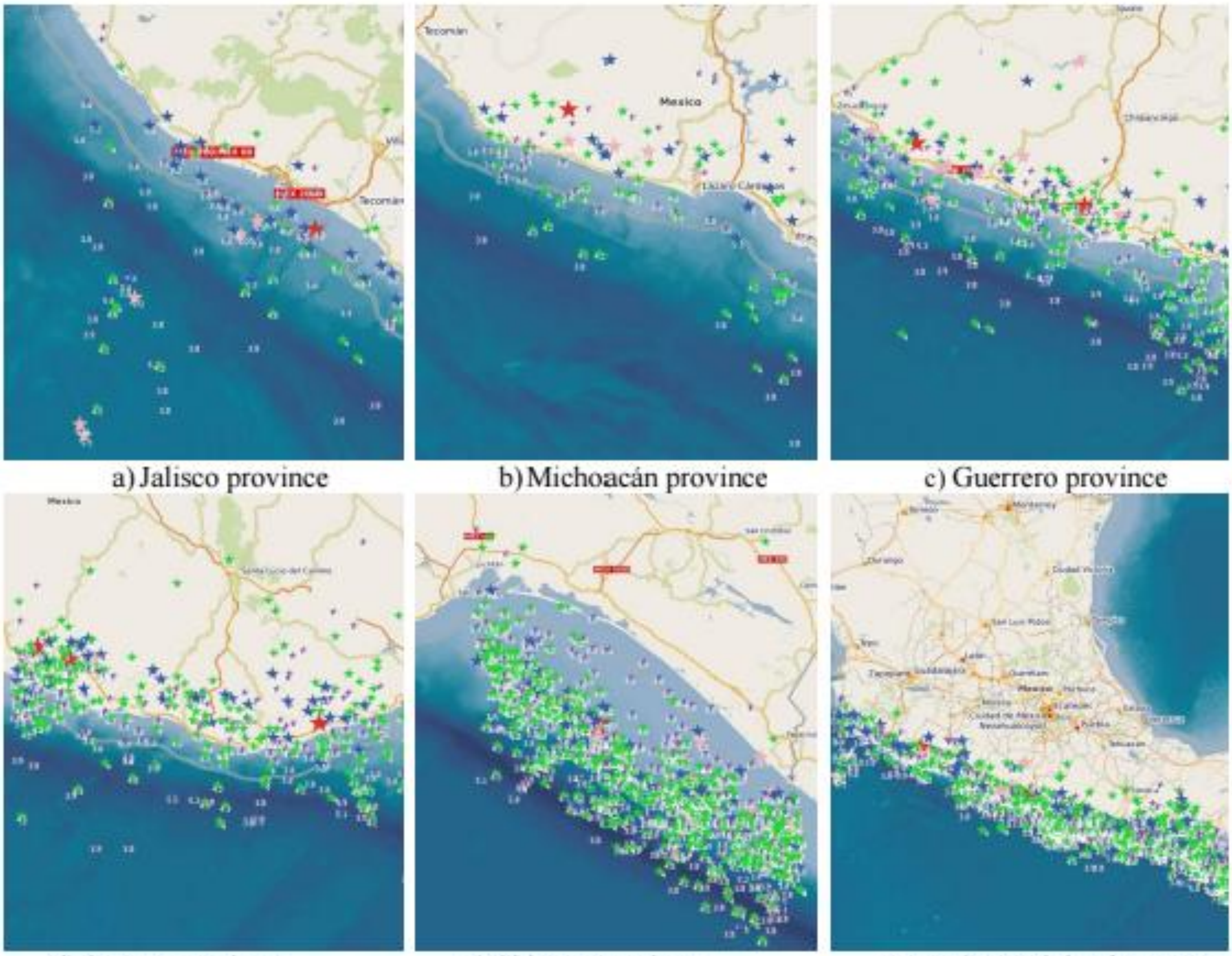
The considered area from Chiapas is limited by the coordinates [13.0°-17.0° N, 92.1°-95.0° W] and depth $h \leq 60$ km, where occurred 1022 seisms with $M_s \geq 3.8$ along 24 years. In this region we have data for 24 years. For each of these years we obtained a GR relationship given by: $\log \dot{N}(M) = a - bM$. In figure 2(e), we show the behaviour of the linear correlation (7). The obtained relation is $a = 3.6346\,b + 1.564$ (with $R = 0.9579$). Also, in this case we observed that the coefficient of b-GR is not so far to 4.

2.6. Global case

Finally, we calculate the linear correlation given by (7) (figure 2(f)) for the global seismicity of the five mentioned seismic provinces obtained the relation: $a = 4.2b + 1.49$ (with $R = 0.8965$), which nevertheless the seismic heterogeneity between the five provinces the coefficient of the b-GR parameter results 4.2 not so far from (7). In summary, we can assert that the linear relationship between the GR coefficients a and b given by (7) [1] is a relevant feature of seismicity.

3. Concluding remarks

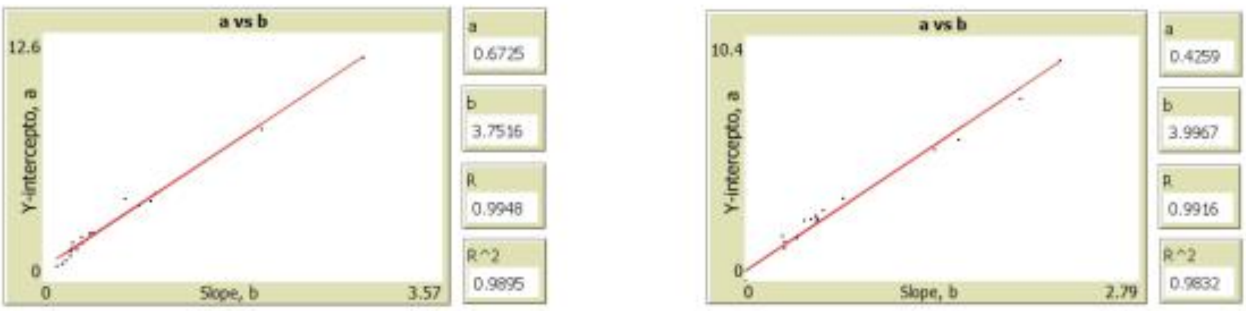
In 2020, we suggested that the a and b-GR parameters are linearly correlated. In that work we arrived at this result by means of three approaches. The first one consisted in a linear regression made over the Bayrak et al. data corresponding to 27 seismically active zones around the world, in this work they reported 27 pairs of a and b-GR values for the studied zones. The linear regression that we made lead to (1) in which the coefficient of the b-value is very near to 4. The second approach consisted in a study on around 150 synthetic seismic catalogs based on the so-called OFC spring-block SOC model of subduction zones. Also, in this case the positive correlation obtained between the a and b-GR parameters resulted highly correlated. The third approach was made by using well known empirical seismic relations taken from the literature with result given by (7); that is, the coefficient of the b parameter resulted equal to 4 (very near to the Bayrak et al. data). In the present work we made linear regressions for five seismic provinces of the Mexican Pacific coast, and we find that for the five provinces and also for the whole cumulated data the coefficient of the b-GR parameter is reasonably near to 4. Therefore, we believe that these results are a first confirmation of the validity of (7) that we published in 2020.



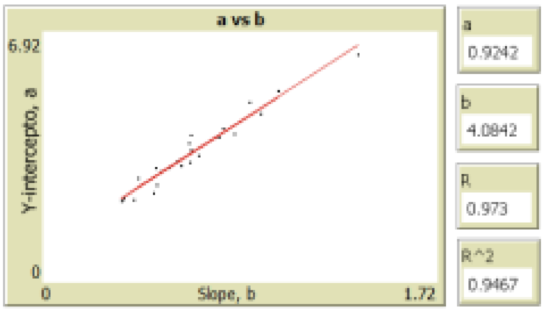
a) Jalisco province b) Michoacán province c) Guerrero province

d) Oaxaca province e) Chiapas province f) Mexican subduction coast

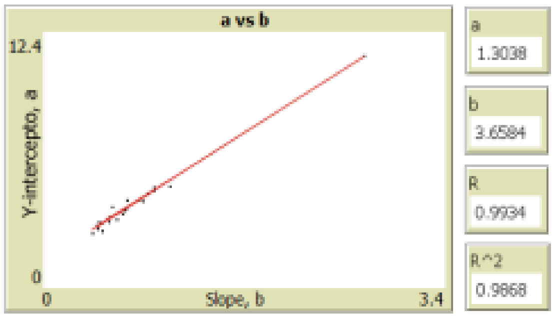
Figure 1: Five seismic provinces of Mexico along the Mexican Pacific coast. The color code for the epicenters is as follows: purple for $3.8 \leq$ magnitude < 4; green for $4.0 \leq$ magnitude < 5; blue for $5.0 \leq$ magnitude < 6; pink for $6.0 \leq$ magnitude < 7; red for $7.0 \leq$ magnitude. (map source: https://www.terrestris.de/)



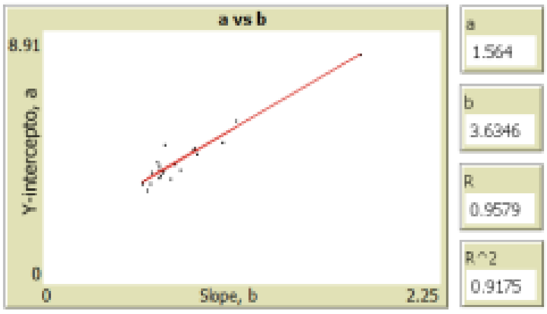
a) Jalisco province b) Michoacán province



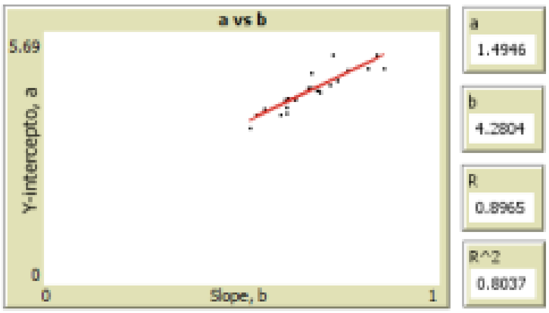
c) Guerrero province



d)Oaxaca province



e) Chiapas province



f) Mexican subduction coast

Figure 2: Linear correlation given by (7) for the five seismic provinces of Mexico and all the Mexican Pacific coast.

4. References

[1] Perez-Oregon J, Muñoz-Diosdado A, Rudolf-Navarro A H, Guzmán-Sáenz A and Angulo-Brown F 2018 *Journal of Seismology* **22** 1025-1035

[2] Gutenberg B and Richter C F 1944 *Bulletin of the Seismological Society of America* **34** 185-188

[3] Gutenberg R and Richter C F 1954 *Seismicity of the Earth and Associated Phenomena* (Princeton, New Jersey: Princeton University Press)

[4] Bayrak Y, Ylmaztürk A and Oztürk S 2002 *Journal of Geodynamics* **34** 653-666

[5] Olami Z, Feder H J S and Christensen K 1992 *Physical Review Letters* **68** 1244-1247

[6] Bak P, Tang C and Wiesenfeld K 1987 *Physical Review Letters* **59** 381-384

[7] Bak P, Tang C and Wiesenfeld K 1988 *Physical Review A* **38** 364-374

[8] Bak P 1996 *How nature works: The science of self-organized criticality* (New York: Springer)

[9] Feder H J S and Feder J 1991 *Physical Review Letters* **66**(20), 2669

[10] Ito K and Matsuzaki M 1990 *Journal of Geophysical Research Solid Earth* **95**(B5), 6853–6860

[11] Geller R J, Jackson D D, Kagan Y Y and Mulargia F 1997 *Science* **275**(5306), 1616

[12] Salinas-Martínez A, Perez-Oregon J, Aguilar-Molina A M, Muñoz-Diosdado A and Angulo-Brown F 2023 *Entropy*, **25**(5), 816

[13] Ruff L and Kanamori H 1980 *Physics of the Earth and Planetary Interiors* **23**(3) 240-252 https://doi.org/10.1016/0031-9201(80)90117-X.

[14] Perez-Oregon J, Muñoz-Diosdado A, Rudolf-Navarro A H and Angulo-Brown F 2020 *Pure and Applied Geophysics* **177**(2), 889-903

[15] Aki K 1981 *American Geophysical Union* **4** 566–575

[16] Brown F A, Guzmán A R, Yepez E, Navarro A R and Miller C P 1998 *Geofisica Internacional* **37**(1)